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Application Note AN131 Original Mar 3, 2000, updated Jun 9,2005

Baseband Pulse Shaping For Improved Spectral Efficiency

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The recent growth in the wireless communications industry has placed a strain on the capacity of such systems. There is only so much RF spectrum available, and the revenue achievable by the providers is directly related to the number of users that can be squeezed into the allotted bandwidth at one time. In this article we describe one method for achieving better spectral occupancy of digitally modulated signals commonly called baseband pulse shaping.

The most common method of describing baseband (i.e. no carrier) data is the non return to zero (NRZ) format shown in Figure 1. In this format a logical one is one level and a logical 0 is the other. The data level only changes when the information transitions from a one to a zero or visa versa. There is no correlation from bit to bit. The first question that can be asked is; 'how much spectrum is required to transmit this format?'. The answer lies in the theoretical formula for the signal power spectral density P(f),

$$P(f) = PT[\sin(\pi fT) / \pi fT]^2$$

where P is the total signal power and 1/T = R = the signal data rate. Figure 2 is a plot of P(f) as obtained from performing a DFT on an actual data stream using the SystemView simulation software. Note two features. First, the PSD is zero at frequencies, which are multiples of R. And second, note how slowly the PSD fall off as a function of frequency. From the basic equation it is easy to show that the PSD fall off as $1/n^2$ where n is the number of the lobe in the spectral display. As a result, the height of the first lobe is down only 13.2 dB from the main peak. This is unacceptable spectrum efficiency for most user applications.

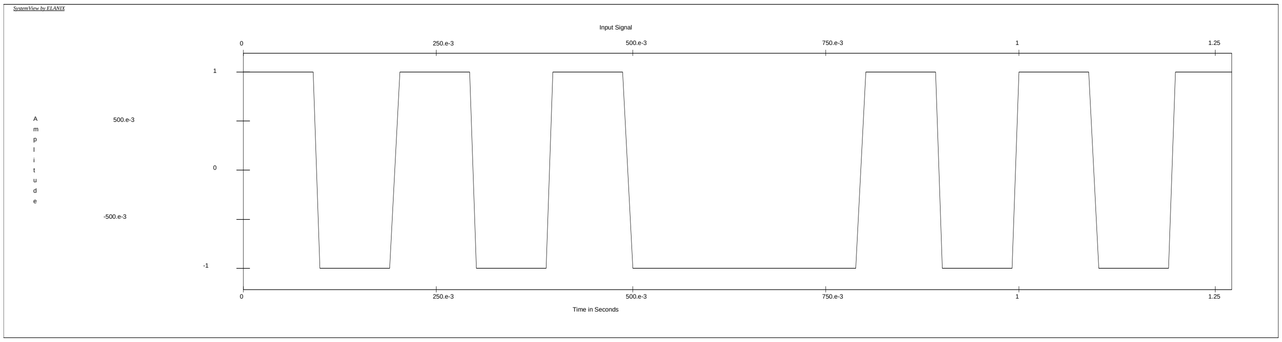

Figure 1. NRZ data waveform

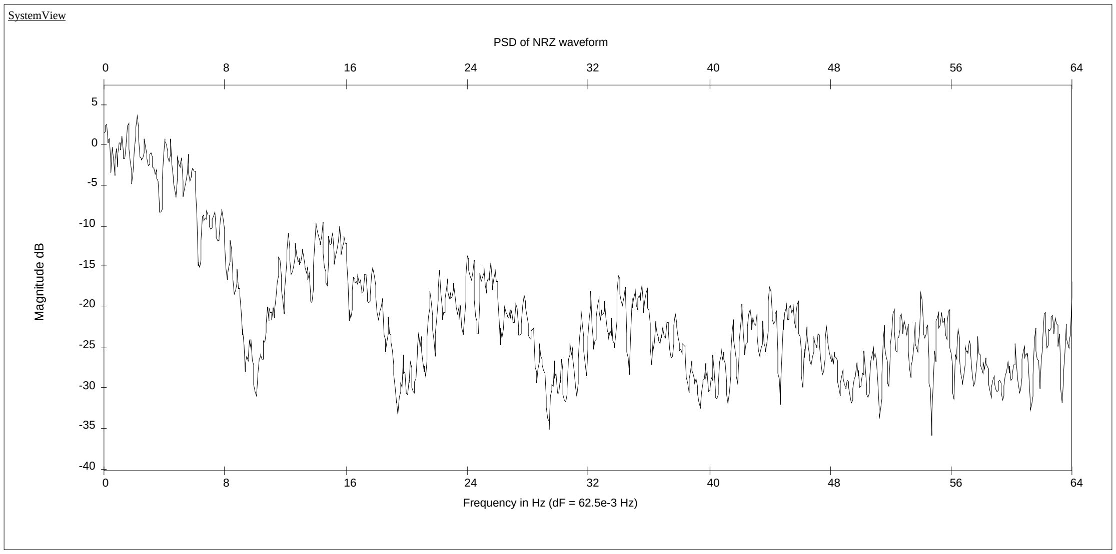

Figure 2. Power spectral density of NRZ waveform

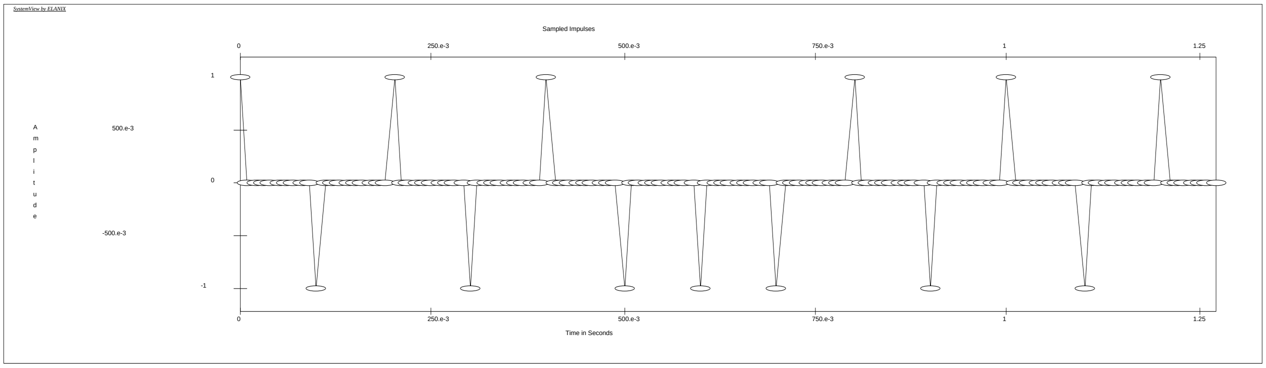

Figure 3. Data impulses

The mathematical description of the NRZ waveform is given by the equation below;

$$
\begin{aligned}
d(t) &= \sum_{k=0} c_k p(t-kT) && c_k = +/-1 \\
p(t) &= 1 && 0 \le t < T \\
&= 0 && \text{otherwise}
\end{aligned}
$$

It is instructive to rewrite the above equation into a slightly more complicated but more useful form;

$$d(t) = \sum_{k=0} c_k \delta(t-kT) * p(t)$$

where δ(t) is the unit impulse function and * indicates the convolution operation. What this equation says is that the NRZ waveform can be developed by impulsing a filter whose response is p(t). Thus if we drive the p(t) filter with the impulse train of figure 3, the NRZ waveform of Figure 1 results. This concept lets us describe other forms of pulse shaping filters on an equal basis.

The theoretically ideal pulse shaping filter has long been known. Nyquist showed that a filter of the form;

$$p_n(t) = \sin[\pi t / T] / [\pi t / T] \equiv \mathrm{sinc}[t / T]$$

is the desired result. The spectral occupancy of this pulse is obtained via Fourier transforms with the neat result;

$$H_n(f) = 1 \qquad 0 \le f \le 1/2T$$
$$= 0 \qquad \text{otherwise}$$

Note the reversal of the time and frequency functions between the Nyquist pulse and the NRZ pulse.

Of course a p(t) can be chosen which has an arbitrarily narrow spectral occupancy. However, the filter output after driving it with the data impulses washes everything together, and thus it is generally impossible to separate out the data after the filter. This is the concept known as inter symbol interference (ISI). Observe that for a Nyquist pulse,

$$p_n(kT) = 0 \quad k \neq 0$$

What this means is that if we sample the output of the filter at times t = kT, the input data impulse at the input is exactly recovered. The Nyquist pulse has no ISI. A convenient way of visualizing ISI is the so called eye diagram. In the eye diagram, the output signal is 'folded' back onto itself modulo T sec. An oscilloscope with a trace time of T seconds and set in a persistence mode will generate the eye diagram. Figure 4 shows such a diagram for a data stream of the type described. The sharp cross over point in the middle is the ideal sampling point. The fact that there is only one value (+/-) indicates no ISI.

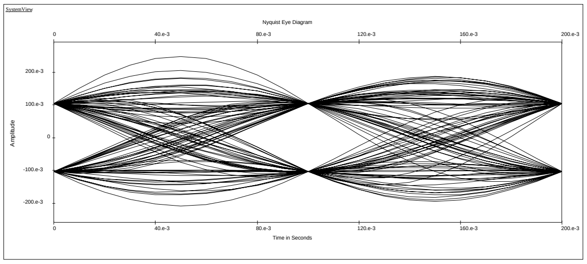

Figure 4. Nyquist pulse eye diagram

If that was all there was to the issue, this article would not be necessary! But there is more. First of all it should be pointed out that the Nyquist pulse does not exist in the real world. This is because mathematically exists for all time which is physically impossible to implement. What must be done is to limit the time extent of the response. The problem with that is the slow ~1/t fall off of the time function. The actual PSD of the shortened filter does not behave very well without a very large number of data points which increases the computational time to produce an output. Figure 5 shows the actual PSD of a Nyquist pulse time shortened to +/- 5T. It doesn't much look like the ideal brick wall filter that is desired.

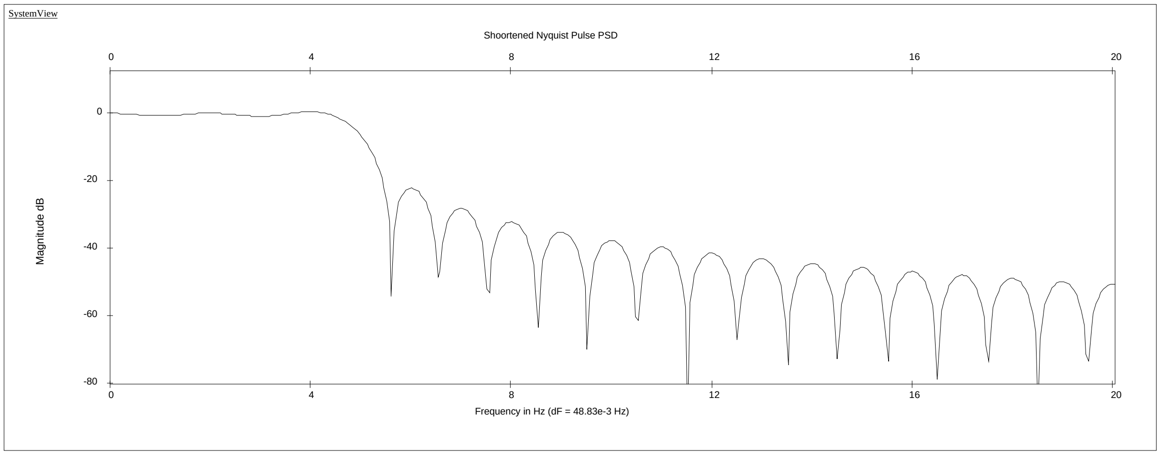

Figure 5. Power spectra of truncated sinc pulse

A popular variant of the Nyquist filter is the raised cosine (rc) filter defined by the relations,

$$p_{rc}(t) = \mathrm{sinc}(t / T)\cos(\pi\beta t / T) / [1 - 4\beta^2 t^2 / T^2]$$

$$\begin{aligned} H(f)_{rc} &= 1 && 0 \le f < (1 - \beta) / 2T \\ &= .5[1 - \sin\{\pi T(f - 1 / 2T)\beta\}] && (1 - \beta) / 2T \le f \le (1 - \beta) / 2T \\ &= 0 && \text{otherwise} \end{aligned}$$

The factor β is called the roll off factor or the access bandwidth. A value of β = 0 reduces rc pulse to the Nyquist pulse. The raised cosine filter time response falls off much faster in time ~ 1/ than the Nyquist pulse, and still has the nice property of no ISI.

There is another issue to discuss when describing a complete communication system. The development thus far has only dealt with the generation or transmission of the signal. What are the implications when we build the corresponding receiver? Here we introduce the concept of a ‘matched filter’. The matched filter is the optimum receiving filter when the shape p(t) of the signal is known. It maximizes the SNR at the sample time when the data is recovered. The important fact is that the shape of the matched filter is identical to the shape of the transmitting filter (hardly a surprise!). Figure 6 shows a SystemView block diagram of a simple communications system with the concepts developed to this point. It is an optimum system in terms of detecting the transmitted signal in noise or equivalently producing the minimum bit error rate for the amount of signal power expended.

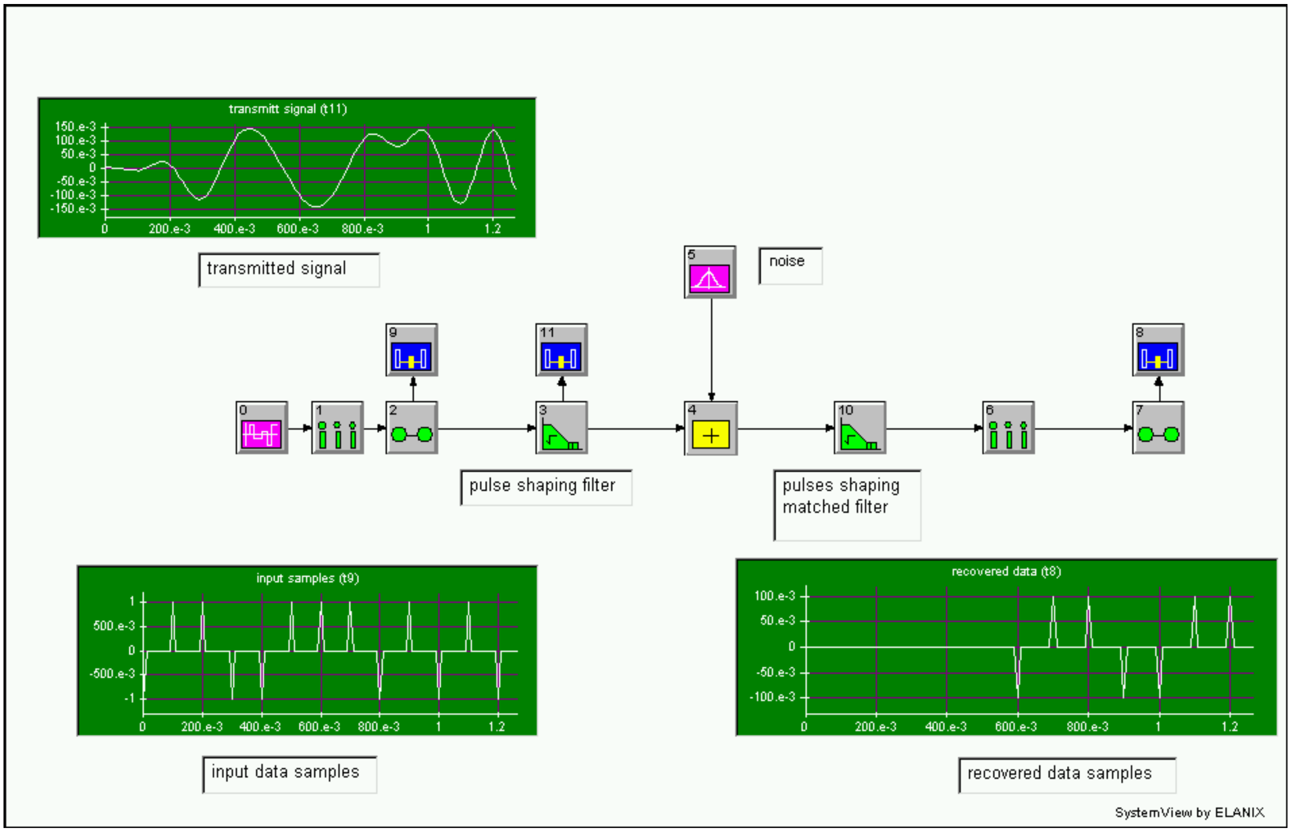

Figure 6 Simple model of an ideal communication system

There is still one more chapter to this story. The issue of ISI must be revisited. We showed that the raised cosine filter does not have ISI at the transmit side, but the output of the matched filter y(t) is actually the convolution of the impulse response with itself, or

$$y(t) = p_{rc}(t) * p_{rc}(t)$$

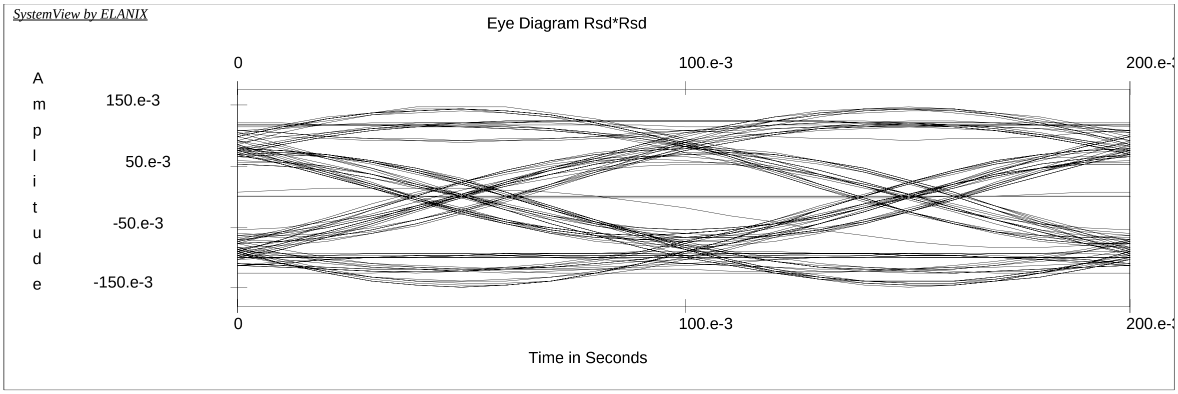

Figure 7 Eye diagram of correlated raised cosine filter

In Figure 7 the eye diagram of the received waveform y(t) is shown. Note that ISI has been reintroduced into the detection process. It can be shown that neither the NRZ nor the Nyquist pulses, which have no ISI at the transmitter, have ISI introduced in the matched filter detection path.

A neat variation of all of this is the root raised cosine (rrc) filter. The idea is simple. Put half of the raised cosine filter in the transmitter, and the other half in the receiver. Mathematically the impulse response, , is given by,

$$p_{rc}(t) = p_{rrc}(t) * p_{rrc}(t)$$

$$p_{rrc}(t) = \{(4b / T)\cos[D_{+} t] + \sin[D_{-} t] / t\} / [(4pbt / T)^{2} - 1]$$
$$D_{-} = (1-b)p / T$$

In the frequency domain the equivalent expression is;

$$H_{rc}(f) = H^{2}_{rrc}(f)$$
or
$$H_{rrc}(f) = \sqrt{H_{rc}(f)}$$

which gives rise to the name ‘root’ raised. In this way the transmit and receive filters are matched to each other, and there is no ISI introduced in the detection process.

Figure 8 shows the impulse response of the sinc, rc, and rrc filters. The rc and rrc are nearly identical and fall off much faster than the sinc pulse. Table 1 summarizes the ISI characteristics of the pulse shaping filters described in this report at the transmission and after the matched filter reception. Figure 9 shows the eye diagram of the rrc filter showing the ISI at the transmit side. However, Figure 10 shows the eye diagram after the matched filter correlation which shows no ISI. Figure 10 is also the picture that would be obtained if we looked at the eye diagram of the rc filter after transmission.

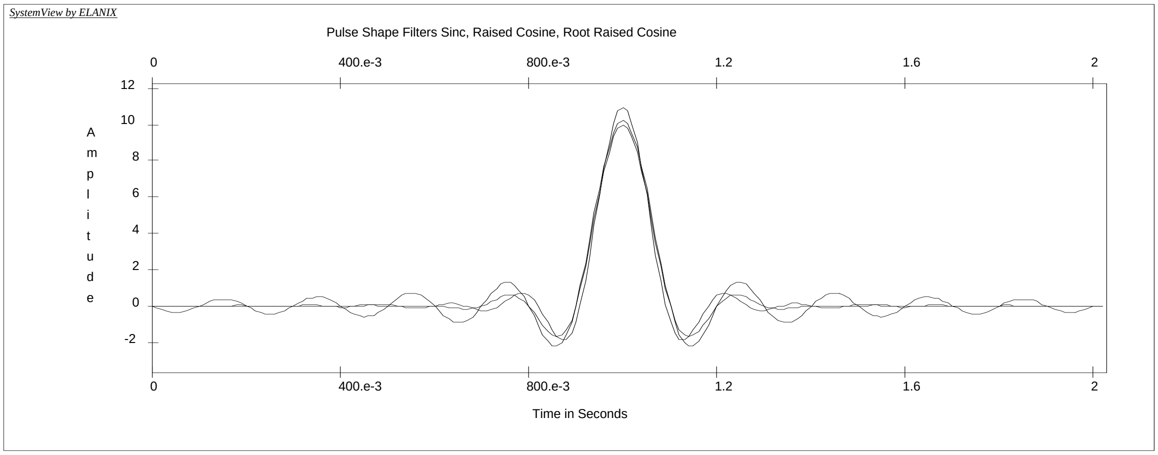

Figure 8 Pulse shaping filter time responses

Pulse Shape	ISI at Transmitter	ISI at Receiver
Rectangular (NRZ)	no	no
Sinc(Nyquist)	no	no
Raise Cosine	no	yes
Root Raised Cosine	yes	no

Table 1 Summary of pulse shaping filter ISI characteristics

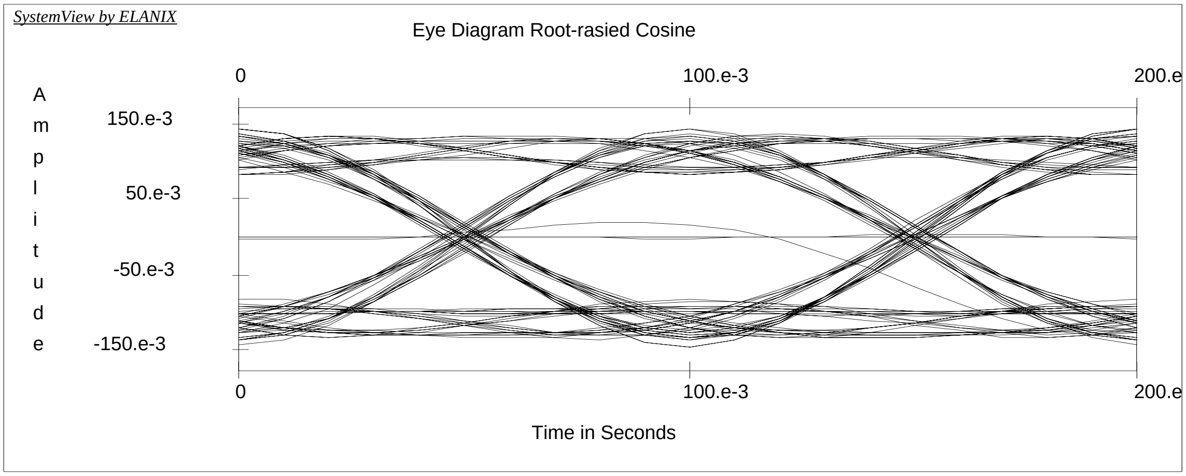

Figure 9. Eye diagram of root raised cosine filter

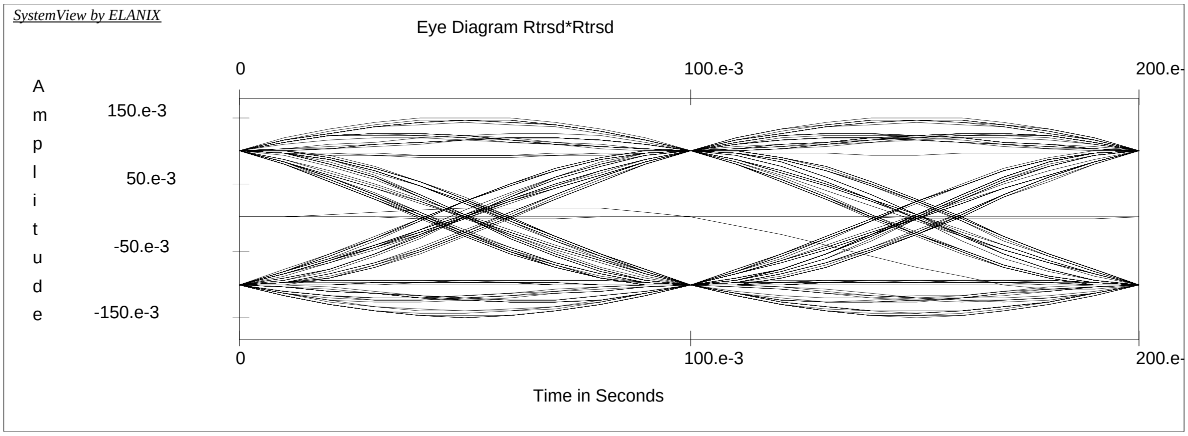

Figure 10. Eye diagram of correlated root raised cosine filter

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