4.1 REGRESSION ANALYSIS

Regression analysis is a technique whereby we obtain a system of equations, solve it, and get the values of the coefficients for a best curve fit.

Mathematical Basics of Linear Curve Fitting

Let's assume we made 'N' measurements y_i at the stimulating points x_i. I.e. we obtained the array $\{x_i , y_i\}$. Subsequently, these measured values were plotted.

A curve Y(x) shall be fitted to this array of measured data points using least square curve-fitting technique.
Referring to an individual measurement point, the fitting error is:

$$E_i = Y(x_i) - y_i \tag{1}$$

and for all data points:

$$E = \sum_{i=1}^{N} E_i^2 = \sum_{i=1}^{N} \left[Y(x_i) - y_i \right]^2 \tag{2}$$

This error shall be minimized.

The fitting will be done by varying the coefficients of the fitting curve of equation (2). The minimum of the total error E depends on the values of these coefficients. This means, we have to differentiate E partially versus the curve coefficients and to set the results to zero. We obtain a system of equations, solve it, and get the values of the coefficients for a best curve fit. This is known as regression analysis.

NOTE: This regression analysis is simple for a straight line fit. But in general, measured data is non-linear. Unfortunately, a non-linear regression analysis can be quite complicated. This problem can be solved if we use a suitable transformation on the measured data. This means that the measured data is transformed to a linear context between the y_i- and the x_i-values. As will be seen in the diode example later, this is a pretty smart way to get the curve fitting parameters easily without much calculations.

LINEAR CURVE FITTING

IC-CAP File:
$ICCAP_ROOT/examples/demo_features/4extraction/basic_PEL_routines/1fit_lin.mdl

Provided we have got an array of N measured data points of the form $\{x_i,y_i\}$.
A linear curve with the equation

$$y(x) = m\,x + b \qquad (3)$$

shall be fitted to these points. This situation is depicted below.

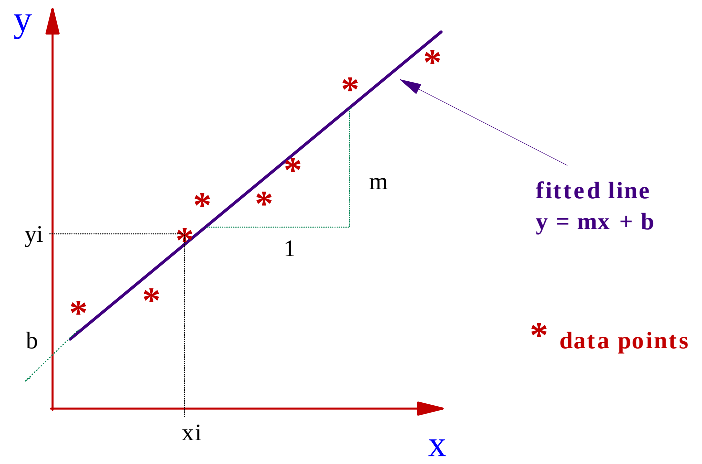

Fig_1: linear regression applied to measurement points

The error of the i-th measurement is:

$$E_i = [\,m\,x_i + b\,] - y_i \qquad (4a)$$

Using the least means square method following equ.(2) yields:

$$E = \sum_{i=1}^{N} E_i^2 = \sum_{i=1}^{N} [\,m\,x_i + b - y_i\,]^2 \overset{!}{=} \text{Minimum} \qquad (4b)$$

Partial differentiation versus slope 'm' gives:

$$2 \sum_{i=1}^{N} [\,m\,x_i + b - y_i\,]\,x_i = 0 \qquad (5)$$

and versus y-intersect 'b':

$$2 \sum_{i=1}^{N} [\,m\,x_i + b - y_i\,] = 0 \qquad (6)$$

We obtain from (5) after a re-arrangement:

$$m \sum_{i=1}^{N} x_i^2 + b \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} y_i x_i \qquad (7)$$

and from (6):

$$m \sum_{i=1}^{N} x_i + N b = \sum_{i=1}^{N} y_i \qquad (8)$$

Multiplying (7) by -N and (8) by $\Sigma\, x_i$ and adding these two equations allows the elimination of the coefficient 'b', and we can separate the slope 'm':

$$m \left[\left(\sum_{i=1}^{N} x_i \right)^2 - N \sum_{i=1}^{N} x_i^2 \right] = \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i - N \sum_{i=1}^{N} x_i y_i \qquad (9)$$

or:

$$m = \frac{\sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i - N \sum_{i=1}^{N} x_i y_i}{\left(\sum_{i=1}^{N} x_i \right)^2 - N \sum_{i=1}^{N} x_i^2} \qquad (10)$$

and from (8) for the y-intersect 'b':

$$b = \left[\sum_{i=1}^{N} y_i - m \sum_{i=1}^{N} x_i \right] / N \qquad (11)$$

with 'm' according to (10).

With equations (10) and (11), we determined the values of the two coefficients of the linear curve which fits best into the 'cloud' of measured data.
Finally, a curve fitting quality factor r^2 is defined. Its value ranges from $\{0 < r^2 < 1\}$. The closer it is to 1, the better is the fit of the linear curve.

$$r^2 = m^2 \frac{\sum_{i=1}^{N} x_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} x_i \right)^2}{\sum_{i=1}^{N} y_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} y_i \right)^2} \qquad (12)$$

with 'm' from (10)

Note: For more details on this chapter see the manuals for the scientific calculators of Hewlett-Packard.

